12DD 44 – I (10)

B.Sc. I Semester Degree Examination, Nov./Dec. 2012
MATHEMATICS
Paper – 1.1. : Algebra – I

Time : 3 Hours Max. Marks: 60

Instructions : *1) Answer **all** the Sections.*
*2) Mention the question numbers **correctly**.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Define quantifiers and types of quantifiers.
2. Negate :

   "All odd numbers are not prime numbers and some prime numbers are even".
3. Find the truth set of $P(x) : | x - 1) < 3$ when the replacement set is the set of natural numbers N.
4. Prove that composition of functions is associative.
5. Define countable set.
6. Show that $E = \{2, 4, 6, ...\}$ is countable.
7. Define equivalent matrix.
8. Define eigen value and eigen vector of a square matrix.
9. Find eigen values of matrix $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$.
10. Find the characteristic equation of $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.
11. Define consistent and inconsistent system of linear equations.
12. State Cayley-Hamilton theorem.

SECTION – B

Answer **any three** of the following : **(3×5=15)**

1. Prove that :

   $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$

2. If $P \to (q \wedge r)$, $\sim S \to (\sim q \vee \sim r)$ and P, prove S, by reduction and absurdum.

3. Prove that the following propositions are false by giving counter examples.

   i) $x^2 - 6x + 8 = 0$

   $\forall x \in R$ such that $2 \le x \le 4$

   ii) $AB = 0 \Rightarrow A = 0$ or $B = 0$, where A and B are 2×2 matrices.

4. $T[\sim P(x)] = \{T[P(x)]\}'$.

SECTION – C

Answer **any one** of the following : **(1×5=5)**

1. Prove that every subset of a countable set is countable.

2. If $f : A \to B$ and $g : B \to C$ be two bijective functions then $(g \circ f)^{-1}$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

SECTION – D

Answer **any four** of the following : **(4×5=20)**

1. Find the rank of the matrix A using the elementary row operations where A is given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

2. Find the inverse of the matrix A by elementary transformations where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

3. Find the eigen values and eigen vectors of the matrix.

$$\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

4. Solve completely the system of equations :

$$x_1 + 3x_2 + 2x_3 = 0$$

$$2x_1 - x_2 + 3x_3 = 0$$

$$3x_1 - 5x_2 + 4x_3 = 0$$

$$x_1 + 17x_2 + 4x_3 = 0$$

5. Verify Cayley-Hamilton theorem.
for

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

and hence find A^{-1}.

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12DD 44 – I (11)

B.Sc. I Semester Degree Examination, Nov./Dec. 2012
MATHEMATICS
Paper – 1. 2 – Calculus – I

Time : 3 Hours   Max. Marks : 60

Instructions : *1) Solve **all** the questions.*
*2) Write the question numbers **correctly**.*

PART – A

I. Answer **any ten** questions : **(10×2=20)**

1) If $f(x) = \begin{cases} x-2 & \text{for } x<0 \\ x^3-3 & \text{for } x>0 \end{cases}$

Find $\lim_{x \to 0} f(x)$, if it exists.

2) Define continuity of the function f(x) at x = a and also define the continuity of the function in the interval [a, b].

3) Find the n^{th} order derivative of $y = \log(ax + b)$.

4) Write the formula for the n^{th} order derivatives of $y = e^{ax}\cos(bx + c)$ and $y = e^{ax}\sin(bx + c)$.

5) Show that the curves $r = a\theta$ and $r = a/\theta$ intersect each other orthogonally.

6) Write the formula for the centre of curvature in Cartesian and parametric form with usual notations.

7) Define cusp, node and isolated point of a curve.

8) Find the polar subtangent and polar subnormal for $r = a\cos 2\theta$, at $\theta = \pi/6$.

9) Find the Pedal equation of the curve $r = ae^{\theta\cot\alpha}$.

10) With usual notations show that $\frac{ds}{dr} = \frac{r}{\sqrt{r^2 - p^2}}$.

11) Find the envolope of the family of circles $(x-\alpha)^2 + y^2 = 4\alpha$.

12) Find the asymptotes parallel to the co-ordinate axes for the curve $xy^3 - x^3 = a(x^2 + y^2)$.

P.T.O.

PART – B

II. Answer **any four** questions : **(4×5=20)**

1) If f(x) is a continuous function defined on [a, b] then it attains its bounds.

2) Find the n^{th} order derivative of $\sin^2 x \cos^3 x$.

3) If $x = \sin t$, $y = \cos pt$ show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, x\, y_{n+1} - (n^2 - p^2)\, y_n = 0$.

4) Show that the curves $r^2 = a^2 \cos 2\theta$ and $r = a(1 + \cos\theta)$ intersect at an angle $3\sin^{-1}(3/4)^{1/4}$.

5) Show that the Pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2 b^2}{p^2} + r^2 = a^2 + b^2$.

6) Derive the formula for radius of curvature in polar form.

PART – C

III. Answer **any four** questions : **(4×5=20)**

1) Find the radius of curvature of the cardioide $r = a(1 + \cos\theta)$. Also show that ρ^2 / r is a constant.

2) Find the co-ordinates of centre of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

3) Find the envolope of the family of circles whose centre lies on the parabola $y^2 = 4ax$ and passing through the vertex.

4) Show that the curve $r = \frac{a\theta^2}{\theta^2 - 1}$ has a point of inflexion at $r = \frac{3a}{2}$.

5) Find all the asymptotes of the curve $y^3 + x^2 y + 2xy^2 - y + 1 = 0$.

6) Trace the curve $r = a \sin 3\theta$. (The three leaved rose).
