I Semester B.Sc. Degree Examination, Nov./Dec. 2013
MATHEMATICS
Paper – 1.1 : Algebra – I

Time: 3 Hours Max. Marks: 60

Instruction : *Answer **all** questions.*

SECTION – A

Answer **any ten** of the following : (10×2=20)

1. Define Replacement set and truth set of an open sentence.
2. Define quantifiers and types of quantifiers.
3. Find the truth set of p(x) : $|x - 1| < 3$ if R[p(x)] = N the set of natural numbers.
4. Negate "If all integers are even then some people like logic".
5. Prove that the composition of mappings is associative.
6. Define countable and denumerable sets.
7. Define characteristic equation and characteristics roots of a square matrix.
8. Find the Eigen values of $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$.
9. Prove that if λ is an Eigen value of A then λ^2 is an Eigen value of A^2.
10. Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$$

11. Define consistent and inconsistent system of equations.
12. State Cayley-Hamilton theorem with explanation.

SECTION – B

Answer **any three** of the following : **(3×5=15)**

1. State the rules of inferences with illustration.
2. "If $p \to q$ is true and $\sim q$ is true" then prove that p is false by direct method.
3. Symbolise the following and negate :
   "Some students are lazy or all students are hard working".
4. If p(x) and q(x) be the open sentences with same replacement set then prove that $T[p(x) \wedge q(x)] = T[p(x)] \cap T[q(x)]$.
5. Prove that $T[\sim p(x)] = \{T(p(x))\}'$.

SECTION – C

Answer **any one** of the following : **(1×5=5)**

1. If $f : X \to Y$ be a mapping and if C and D are any two subsets of Y, then prove the following :
   i) $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$
   ii) $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$.
2. Show that the sets $N \times N$ and $Z \times Z$ are equivalent.
3. Prove that the set $N \times N$ is denumerable.

SECTION – D

Answer **any four** of the following : **(4×5=20)**

1. Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ -1 & -2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

2. Find the rank of the following matrix A by reducing it to normal form where

$$A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}.$$

3. Find the inverse of the matrix A by elementary transformations where

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

4. Solve completely the following system of equations :

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0.$

5. Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 4 & -1 \\ 1 & +2 \end{bmatrix}.$$

6. Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}.$$

B.Sc. I Semester Degree Examination Nov./Dec. 2013
MATHEMATICS
Paper : 1.2 – Calculus – I

Time : 3 Hours | Max. Marks : 60

Instructions : *1) Answer **all** the Sections.*
*2) Write the question numbers **correctly**.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. If $f(x) = \begin{cases} x^2+3, & \text{if } x \le 1 \\ x+1, & \text{if } x > 1 \end{cases}$, find $\lim_{x\to1} f(x)$, if it exists.
2. Define removable discontinuity and ordinary discontinuity.
3. Find the n^{th} derivative of $y = e^{ax} \sin(bx + c)$.
4. Find the ratio of polar sub-tangent and polar sub-normal for the curve $r = ae^{b\theta^2}$.
5. Show that for the curve $r = a\theta$ is $p = \frac{r^2}{\sqrt{r^2+a^2}}$.
6. Find $\frac{ds}{dx}$ and $\frac{ds}{dy}$ for the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
7. Show that the curve $y = e^{-x}$ concave upwards every where.
8. Find the radius of curvature of the curve $y = 4 \sin x - \sin 2x$ at $x = \pi/2$.
9. Write the formula for the co-ordinates of the centre of curvature and the equation of circle of curvature.
10. Find the envelope of family of the circle $(x - \alpha)^2 + y^2 = 4\alpha$, where α is a parameter.
11. Find the singular point on the curve $x^3 + x^2 + y^2 - x - 4x + 3 = 0$.
12. Find the asymptotes parallel to the co-ordinate axes for the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

P.T.O.

SECTION – B

Answer **any four** of the following : **(4×5=20)**

1. If f(x) is continuous in [a, b], f(a) and f(b) have opposite signs then f(x) vanishes for atleast one value of x in [a, b].
2. Find the n^{th} derivative of $y = \sin^2 x \cos^3 x$.
3. If $y = (\sin^{-1} x)^2$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$.
4. Define pedal equation of a curve and with usual notations prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.
5. Find the angle of intersection of two curves $r = a \log \theta$ and $r = \frac{a}{\log \theta}$.
6. Find the envelope of the family of the circle $x^2 + y^2 - 2ax \cos \alpha - 2y \sin \alpha = c^2$ where α is a parameter.

SECTION – C

Answer **any four** of the following : **(4×5=20)**

1. Show that the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1) is $\sqrt{2}/3$.
2. Find the point of inflexion on the curve $x = \log\left(\frac{y}{x}\right)$.
3. Find the circle of curvature for the curve $xy = c^2$ at (a, c).
4. Find the evolute of the curve $x^2 - y^2 = a^2$.
5. Find all the asymptotes of the curve $x^2y^2 = a^2 (x^2 + y^2)$.
6. Trace the curve cissoid $y^2 (a - x) = x^3$, $a > 0$.