



27123 (New)

B.Sc. I Semester Degree Examination, Oct./Nov. - 2018

MATHEMATICS

Algebra - I

Paper - 1

(New)

Time : 3 Hours

Maximum Marks : 80

Instructions to Candidates:

1. Part A - All questions are compulsory.
2. Part B - Solve any **Five** questions from **Seven** questions.
3. Write the question number correctly.

PART-A

I. Answer the following questions.

(10×2=20)

1. Prove that $\frac{\cos \theta + i \sin \theta}{\cos \phi + i \sin \phi} = \cos(\theta - \phi) + i \sin(\theta - \phi)$.
2. Find the fifth roots of unity.
3. Solve the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ given that it has a root $2 + \sqrt{3}$.
4. Find the quotient and remainder when $4x^4 - 3x^2 + 4x - 2$ is divided by $x + 2$.
5. Find the equation whose roots are reciprocals of the equation $3x^4 - 3x^2 + 4x - 1 = 0$.
6. Transform the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another. Whose leading coefficient will be unity.

7. Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$

[P.T.O.]



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8. Verify the system of equations for consistent given below

$$x + y - 2z = 5$$

$$x - 2y + z = -2$$

$$-2x + y + z = 4$$

9. Define normal and echelon form of the matrix.

10. Find the eigen values of matrix $A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$.

PART - B

Answer any FIVE complete questions.

(5×12=60)

- II. 11. State De - Moivre's theorem and simplify $\left(\frac{\sin \theta + i \cos \theta}{\cos \theta + i \sin \theta} \right)^4$.

12. If $x + \frac{1}{x} = 2 \cos \theta$ then show that $x^n + \frac{1}{x^n} = 2 \cos n\theta$.

- III. 13. Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ whose roots are in A.P.

14. Solve $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing the second term.

- IV. 15. Transform the equation $x^5 + \frac{4}{3}x^4 + \frac{2}{9}x^3 + \frac{1}{12}x^2 + \frac{1}{36} = 0$ into another with integral coefficients.

16. Solve the equation $x^3 - 18x - 35 = 0$ by Cardan's method.

- V. 17. Solve $x^3 - 9x + 1 = 0$ by trigonometric method.

18. Solve the biquadratic equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$



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VI. 19. Define rank of the matrix and find the rank of A

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

20. Find the matrices P and Q such that PAQ is in normal form where

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 3 \end{bmatrix}$$

VII. 21. Find the inverse of the matrix A by the elementary transformations where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

22. Find the values of λ for which the system has a non-trivial solution.

$$3x + y - \lambda z = 0$$

$$4x - 2y - 3z = 0$$

$$2\lambda x + 4y - \lambda z = 0$$

VIII. 23. State and prove Caley - Hamilton theorem.

24. Find the eigen values and eigen vectors of the matrix A.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$