13MY 44 – II (10)

B.Sc. II Semester Degree Examination, May 2013
MATHEMATICS
Paper – 2.1 : Algebra II

Time : 3 Hours Max. Marks : 60

Instruction : *Answer* ***all*** *Sections.*

SECTION – A

I. Answer **any ten** of the followings : **(10×2=20)**

1) State factor theorem.

2) Solve $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ given that if has a root $2 + \sqrt{3}$.

3) Find the quotient and the remainder of $3x^3 - 4x^2 + 2x + 1$ by dividing $x - 3$.

4) Transform the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another, whose leading coefficient will be unity.

5) Define Infimum and suprimum of the sequence.

6) The sequence $\left\{1 - \frac{1}{n}\right\}$ is a monotonic increasing sequence.

7) Show that

'O' is a limit point of the sequence $\left\{\frac{1}{n}\right\}$

8) Define monotonic sequence.

9) Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)} + ...$

10) State limit form of comparison test.

11) State P-series.

12) Discuss the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + ...$

P.T.O.

SECTION – B

II. Answer **any two** of the following : **(2×5=10)**

1) Increase the roots of the equation $4x^4 + 32x^3 + 83x^2 + 73x + 21 = 0$ by 2 and hence solve the equation.

2) Show that the equation :

   $x^{12} - x^4 + x^3 - x^2 + 1 = 0$ has at least four imaginary roots by Descrarate's rule of signs.

3) Solve : $x^3 - 27x + 54 = 0$ by Cardans method.

4) Solve $x^3 - 9x + 1 = 0$ by trigonometric method.

SECTION – C

III. Answer **any three** of the following : **(3×5=15)**

1) Prove that the limit of a convergent sequence is unique.

2) If in a convergent sequence a finite number of terms are removed, the convergence of the sequence will not alter.

3) Test the convergence of the following sequences :

   1) $\left(1+\frac{a}{n}\right)^{n/b}$ 2) $\frac{n+(-1)^n}{n}$

4) If $\{a_n\}$ is a convergent sequence of positive terms then evaluate

   $\lim_{n\to\infty} a_n$ where $a_{n+1} = \frac{4}{2+a_n}$.

5) Show that the sequence $\{x_n\}$ defined $x_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$ is convergent.

SECTION – D

IV. Answer **any three** of the following : **(3×5=15)**

1) State and prove D'Alemberts ratio test.

2) If $\sum U_n$ and $\sum V_n$ be the two series of positive terms such that

   i) $\sum V_n$ is convergent and

   ii) $U_n \leq K\ V_n$, $\forall n$ except perhaps for the finite number of terms in the beginning, where $K > 0$ then prove that $\sum U_n$ is also convergent.

3) Discuss the convergence of the series

i) $\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+.....$

ii) $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+.....$

4) Test the convergence of the series

1) $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\frac{1.2.3.4}{3.5.7.9}+....$

2) $\frac{2!}{3}+\frac{3!}{3^2}+\frac{4!}{3^3}+.....$

5) State Cauchys root test and hence test the convergence of the series $\sum\left(\frac{nx}{n+1}\right)^n$.

B.Sc. II Semester Degree Examination, May 2013
Paper – 2.2 : MATHEMATICS
CALCULUS – II

Time : 3 Hours

Max. Marks : 60

Instructions : *i) Answer **all** the questions.*
ii) Mention the question numbers correctly.

SECTION – A

I. Answer **any ten** of the following : **(10×2=20)**

1) Evaluate $\int \frac{dx}{2x^2 - 2x + 1}$.

2) Obtain the reduction formula for $\int \tan^n x dx$.

3) Evaluate $\lim_{n \to \infty} \sum_{r=1}^{n-1} \frac{1}{\sqrt{n^2 - r^2}}$.

4) Show that $\int_0^{\pi/2} \sin^4 x \cos^2 x dx = \pi/32$.

5) Evaluate $\int_0^{2\pi} \sin^7 \left(x/4\right) dx$.

6) Show that $\int_0^{\pi/4} \sec^4 x dx = 4/3$.

7) Find the length of the arc of the curve $y = \log \sec x$ from $x = 0$ to $x = \pi/3$.

8) Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.

9) The circle $x^2 + y^2 = a^2$ is revolved about the x-axis. Find the volume of the sphere so formed.

10) If $u = \log \left(\frac{x^2 + y^2}{x + y}\right)$ show that $xu_x + yu_y = 1$.

11) Show that the total derivative of $z = xy^2 + x^2y$ where $x = at$, $y = 2at$ is $18a^3t^2$.

12) Define Jacobian of u, v, w with respect to x, y, z.

P.T.O.

SECTION – B

II. Answer **any five** of the following : **(5×5=25)**

1) Evaluate $\int \frac{x}{(x-3)\sqrt{x+1}}\,dx$.

2) Evaluate $\lim_{n\to\infty}\left[\frac{\sqrt{n+1}}{n\sqrt{n}}+\frac{\sqrt{n+2}}{n\sqrt{n}}+\dots+\frac{\sqrt{2n}}{n\sqrt{n}}\right]$.

3) Prove that $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)\,dx$ if $f(2a-x) = f(x)$.

4) Evaluate $\int \frac{x}{(x^2-3x+2)\sqrt{x-1}}\,dx$.

5) Find the area of the curve astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

6) Find the volume of the solid obtained by revolving the cardiode $r = a(1 + \cos\theta)$ about the initial line.

7) Find the surface area of the hemisphere of radius 'a' units.

SECTION – C

III. Answer **any three** of the following : **(3×5=15)**

1) If $u = x^y$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

2) State and prove Euler's theorem on homogeneous functions.

3) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

4) If $x + y + z = u$, $y + z = v$ and $z = uvw$, find the value of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

5) If $u = \log\sqrt{x^2+y^2+z^2}$, show that $(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$.