B.Sc. III Semester Degree Examination, Nov./Dec. 2013
MATHEMATICS
3.1 : Vector Algebra and Solid Geometry

Time : 3 Hours Max. Marks : 60

***Instruction** : Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Define collinear and coplanar vectors.

2. Prove that $\left[\bar{b}\times\bar{c}, \bar{c}\times\bar{a}\ \ \bar{a}\times\bar{b}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$

3. Show that $\sum \bar{a}.\bar{a} = 3$.

4. Find the distance between the points P (5, 4, –6) and Q (9, 8, –10).

5. Find the ratio in which the line joining the points (2, 4, 5) and (3, 5, –4) divides xy plane.

6. Show that the three points (3, 2, –4), (9, 8, –10) and (5, 4, –6) are collinear.

7. Find the direction cosines of the line joining the points (2, –3, 6) and (3, –1, –6)

8. Find the equation of the line passing through the points A (2, 5, 8) and B (–1, 6, 3).

9. Find the angle between the line joining the points (1, 2, 3) and (4, 5, 7) and the plane $x + 3y - 3z = 4$.

10. Find the equation of the plane passing through (0, 1, 6) and parallel to the plane $\bar{r}.(i - 2j) = 3$.

11. Find the projection of the line segment AB on CD. Where A = (1, 3, 5) B = (6, 4, 3), C = (2, –1, 4) and D = (0, 1, 5)

12. Find the volume of the tetrahedron formed by the points (1, –1, 1), (0, 1, 2), (3, 1, 0) and (1, 4, –1).

P.T.O.

SECTION – B

Answer **any two** of the followings : **(2×5=10)**

1. If $\vec{a} = (2, 3, -1)$, $\vec{b} = (1, -1, 2)$ and $\vec{c} = (-1, 2, 2)$ Find $\vec{a} \times (\vec{b} \times \vec{c})$.

2. Show that $\nu \times (\vec{a} \times i) + j \times (\vec{a} \times j) + k \times (\vec{a} \times k) = 2\vec{a}$.

3. Find the unit vector co-planar with $\vec{b}$ and $\vec{c}$ but perpendicular to $\vec{a}$. Where

   $\vec{a} = i - 2j + k$, $\vec{b} = 2i + j + k$, $\vec{c} = i + 2j - k$.

4. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$.

SECTION – C

Answer **any three** of the following. **(3×5=15)**

1. Find the co-ordinates of the points which divides the line joining the points $(1, 3, -4)$ and $(4, 1, -2)$ in the ratio $3 : -1$.

2. Find the direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $2\,lm + 2\,ln - mn = 0$

3. Find the angle between the planes

   $7x + 4y + 4z + 3 = 0$ and

   $2x + y + 2z + 2 = 0$

4. Find the values of 'a' and 'b' such that A (a, 1, 1), B (1, b, –1) and C (1, 3, –3) are collinear

5. Find the refexion of the point (1, –1, 0) in the line. $\dfrac{2x-4}{2} = \dfrac{y-1}{2} = \dfrac{z+3}{-1}$.

SECTION – D

Answer **any three** of the followings : **(3×5=15)**

1. Find the equation of the plane containing the line $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-4}{-2}$ and the point (0, 6, 0).

2. Derive the condition for the three points to be collinear.

3. Find the length of the perpendicular from the point (5, 4, –1) to the line $\frac{x-1}{2}=\frac{y}{9}=\frac{z}{5}$.

4. Find the symmetrical form of the line of intersection of the planes $2x+3y+5z-1=0$, $3x+y-z+2=0$.

5. Find the lengths of the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-6}{4}=\frac{z-5}{5}.$$

B.Sc. III Semester Degree Examination, Nov./Dec. 2013
MATHEMATICS
Paper – 3.2 : Differentiability, Line and Multiple Integrals

Time : 3 Hours Max. Marks : 60

Instructions : *1) Answer* ***all*** *the Sections.*
2) Write the question number ***correctly****.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. State Rolle's theorem and give reasons for inapplicable of the same for the function $f(x) = x^3$ in [1, 2].
2. Verify the Lagrange's mean value theorem for the function f(x) = logx in [1, e].
3. Using Maclaurin's series expand sinx up to the terms containing x^5.
4. Evaluate $\lim\limits_{x\to\pi/2} \dfrac{\log(\sin x)}{(\pi/2 - x)^2}$
5. Evaluate $\lim\limits_{x\to\pi/2} \dfrac{\tan x}{\tan 3x}$.
6. Let f(x) = x for all $x \in [0, 1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be a partition of [0, 1] compute L (p, f).
7. Show that a constant function is R-integrable.
8. State the fundamental theorem of integral calculus and show that $\int_1^2 x^3 dx = \frac{15}{4}$ using the same.
9. Compute the integral $\int_c xy dx$ along the arc of the parabola $x = y^2$ from (1, –1) to (1, 1).

10. Evaluate $\int_0^1 \int_0^4 xy\,dx\,dy$.

11. Evaluate $\int_0^1 \int_0^2 (x+y)\,dy\,dx$.

12. Show that $\int_0^1 \int_0^2 \int_1^2 x^2 yz\,dz\,dy\,dx = 1$.

SECTION – B

Answer **any two** of the following : **(2×5=10)**

1. If f is defined on [1, 2] by f(x) = 3x + 1, $\forall$ x$\in$ [1, 2] then f$\in$R [1, 2] and $\int_1^2 f(x)dx = 11/2$.
2. If f is a monotonic function on [a, b] then f is R-integrable on [a, b].
3. State and prove the necessary and sufficient condition for a bounded function defined on [a, b] to be R-integrable.

SECTION – C

Answer **any three** of the following : **(3×5=15)**

1. State and prove Cauchy mean value theorem.
2. Apply mean value theorem to show that $x > \log(1+x) > x - \frac{x^2}{2}$, $x > 0$.
3. Obtain the Maclaurin's expansion of log (1 + x) and hence deduce that $\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$
4. Evaluate :

   1) $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x\tan x}\right)$ 2) $\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$

5. Evaluate :

1) $\lim_{x \to \infty} 2^x . \sin \frac{a}{2^x}$ 2) $\lim_{x \to 0} \left(\frac{2x+1}{x+1} \right)^{x-1}$.

SECTION – D

Answer **any three** of the following : **(3×5=15)**

1. Let C be any path leading from the origin to the point (1, 1, 1) show that

$$\int_C [2xydx + (x^2 + 2yz)\, dy + (y^2 + 1)\, dz] = 3.$$

2. Evaluate $\iint x^2 y^2 dxdy$ over the + ve quadrant of the circle $x^2 + y^2 = 1$.

3. Evaluate $\iint_D \frac{x^2 y^2}{x^2 + y^2} dxdy$, where D is the annular region between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 1$ by changing to polar co-ordinates.

4. Evaluate $\iiint_R (x - 2y + z)\, dxdydz$

Where R is the region determined by $0 \le x \le 1,\ 0 \le y \le x^2,\ 0 \le z \le x + y$.

5. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$ and $z = 0$.

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