13MY 44 – IV (30)

B.Sc. IV Semester Degree Examination, May 2013
Paper – 4.1 : MATHEMATICS
Abstract Algebra and Linear Algebra

Time: 3 Hours Max. Marks: 60

Instruction : *Answer **all** the Sections.*

SECTION – A

I. Answer **any ten** of the following : **(10×2=20)**

1) Define cyclic group and normal subgroup.
2) Define quotient or factor group.
3) Define homomorphism and isomorphism.
4) Define permutation group.
5) Find the number of generators of the cyclic group of order 60.
6) Define integral domain.
7) Define skew field and field.
8) Define vector space over the field of real numbers.
9) Show that the vectors $\{(1, 1-1)\ (2, -3, 5), (-2, 1, 4)\}$ are linearly independent.
10) Define Basis and Dimension.
11) Define linear transformation.
12) State Rank-Nullity theorem.

SECTION – B

II. Answer **any three** of the following : **(3×5=15)**

1) Define right and left cosets of H in G. Hence state and prove Lagranges theorem.
2) Prove that the intersection of two normal subgroups is again a normal subgroup.
3) State and prove fundamental theorem of homomorphism.
4) Prove that the set G/H of all cosets of a normal subgroup H of a group G is a group under the binary operation defined by $H_a \cdot H_b = H_{ab},\ \forall\ H_a, H_b \in \frac{G}{H}$.
5) P.T. the centre Z of a group G, is a normal subgroup of G.

P.T.O.

SECTION – C

III. Answer **any two** of the following : (2×5=10)

1) Define a ring and show that the subset S of a ring (R, t, ·) is a subring of R if and only if :

   i) $\forall\, a, b \in S \Rightarrow a + (-b) \in S$

   ii) $\forall\, a, b \in S \Rightarrow a \cdot b \in S$.

2) Show that a ring is without zero divisors if and only if the cancellation laws hold in it.

3) Prove that every finite integral domain is a field.

4) Show that an integral domain with six elements does not exist.

SECTION – D

IV. Answer **any three** of the following : (3×5=15)

1) Prove that the necessary and sufficient conditions for a non-empty subset W of a vector space V(F) to be a subspace of V are $\forall\, a, b \in F$, and $\alpha,\ \beta \in W \Rightarrow a\,\alpha + b\,\beta \in W$.

2) Prove that any two bases of a finite dimensional vector space V have the same finite number of elements.

3) Determine the linear transformation for a matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ for $T : V_3(R) \rightarrow V_2(R)$ relative to the standard basis of $V_3(R)$ and $V_2(R)$.

4) Find the matrix of linear transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (2y - x, y, 3y - 3x)$ relative to the bases $B_1 = \{(1,1)\ (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.

5) State and prove Rank-Nullity theorem.

B.Sc. IV Semester Degree Examination, May 2013
MATHEMATICS
Paper – 4.2 : Differential Equations – I

Time : 3 Hours Max. Marks : 60

Instructions : *i) Answer **all** the questions.*
ii) Mention the question number correctly.

SECTION – A

I. Answer **any ten** of the following : **(10×2=20)**

1) Define general solution and particular solution of a differential equation.

2) Solve the equation $(e^y+1)\cos x dx + e^y \sin x dy = 0$.

3) Find the general solution of the linear differential equation

$\sin x \, {dy}/{dx} + y\cos x = 2 \sin^2 x.\cos x$.

4) Solve $p^2 - 5p + 6 = 0$.

5) Find the complementary function of $(D^4 - 5D^2 + 4)\, y = e^{3x}$.

6) Solve ${d^2y}/{dx^2} - 4y = x^2$.

7) Obtain the general solution of $(D^4 + 1)\, y = \cos x$.

8) Reduce the equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$ to linear deferential equation with constant coefficients and hence find its complementary function.

9) Solve ${dx}/{dt} + wy = 0$, ${dx}/{dt} - wx = 0$

10) Find a part of complementary function of

$\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - (1-\cot x)y = e^x.\sin x.$

P.T.O.

11) Find the Wranskian W for the equation $(D^2 - 1)\, y = \frac{2}{1+e^x}$.

12) Show that the equation $x^2(1+x)\frac{d^2y}{dx^2} + 2x(2+3x)\frac{dy}{dx} + 2(1+3x)y = 0$ is exact.

SECTION – B

II. Answer **any three** of the following : **(3x5=15)**

1) Explain the method of solving the linear differential equation $dy/dx + Py = Q$, where P and Q are functions of x alone.

2) Show that $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$ is exact and hence solve it.

3) Solve $y^2 \log y = xyp + p^2$.

4) Find the general solution and singular solution of $y = px + \sin^{-1}p$.

5) Solve the Bernoulli's equation

$$\sec^2 y\frac{dy}{dx} + x\tan y = x^3.$$

SECTION – C

III. Answer **any three** of the following : **(3x5=15)**

1) Solve the equation $(D^2 + 4D + 4)y = e^{-2x}$.

2) Obtain the general solution of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin 3x$

3) Solve : $(D^3 + 3D^2 + 2D)y = x^2$.

4) Find the general solution of $\frac{d^2y}{dx^2} + 4y = x \sin x$.

5) Solve $x\frac{d^2y}{dx^2} - 2\frac{y}{x} = x + \frac{1}{x^2}$.

SECTION – D

IV. Answer **any two** of the following : **(2×5=10)**

1) Solve $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0$ by the method of changing the independent variable.

2) Solve $x^2\frac{d^2y}{dx^2} - 2(x^2 + x)\frac{dy}{dx} + (x^2 + 2x + 2) = 0$ by reducing it to normal form.

3) Solve $y_2 + y = \sec x$ by the method of variation of parameters.

4) Solve $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$ by finding a part of complementary function.

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