11423

B.Sc IV Semester Degree Examination, May - 2018

MATHEMATICS

Algebra III

Paper - 4.1

Time : 3 Hours Maximum Marks : 60

Instructions to Candidates:

1) Answer **all** the questions.

2) Write the question numbers correctly.

Part - A

Answer any **TEN** questions : **(10×2=20)**

1. Define left and right coset.
2. Define quotient group or factor group.
3. Define centre of a group.
4. Find the order of the elements of the multiplicative group G = {1, -1, i, -i} of fourth roots of unity.
5. Find the number of generators of the cyclic group of order 60.
6. Define normal subgroup and quotient group.
7. Define a vector space over a field F.
8. Define integral domain and division ring.
9. Prove that the intersection of any two subspaces of a vector space V over a field F is again a subspace of V.
10. Define a linear transformation.
11. Define range and kernel of a linear transformation.

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12. Define basis and dimension of a vector space.

Part- B

Answer any **Three** of the following questions. (3×5=15)

13. Define a cyclic group and prove that every subgroup of a cyclic group is cyclic.

14. Let G be a group and H be a normal subgroup of G, then prove that G/H is a homomorphic image of G with H as its kernel.

15. State and prove fundamental theorem of homomorphism of a group.

16. State and prove cayley's theorem.

17. If $f:G \to G'$ be a homomorphism from the group (G, $\bullet$) into (G,*) the prove that

    i) $f(e)=e'$ where e and e' are identity elements of G and G'

    ii) $f(a^{-1})=[f(a)]^{-1} \forall a \in G$

Part - C

Answer any **Two** questions. (2×5=10)

18. Define a ring and show that the subset S of a ring (R,+,$\bullet$) is a subring of R if and only if

    i) $\forall a,b \in S, a+(-b) \in S$

    ii) $\forall a,b \in S, a.b \in S$

19. Prove that a ring is without zero divisors if and only if the cancellation law holds.

20. Define subrings and prove that intersection of any two subrings of a ring is again a subring.

21. Show that a finite integral domain is a field.

Part - D

Answer any **Three** questions from the following : (3×5=15)

22. Prove that a non empty subset w of a vector space V over a field F is a subspace of V if and only if.

    i) $\forall\ \alpha, \beta \in w \Rightarrow \alpha+\beta \in W$

    ii) $C \in F, \alpha \in w \Rightarrow C\alpha \in W$

23. Let V be a vector space over a field F then show that every non empty subset of a linearly independent set of vectors of V is linearly independent.

24. Define basis and dimension of a vector space V over a field F. Show that the set $B = \{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of the vector space $V_3(R)$.

25. Find the matrix of the linear transformation $T: V_2(R) \to V_3(R)$ defined by $T(x,y) = (x+y, x, 3x-y)$ with respect to basis $B_1 = \{(1,1),(3,1)\}, B_2 = \{(1,1,1),(1,1,0),(1,0,0)\}$.

26. Find the range, null space, rank and nullity of the linear transformation $T: V_3(R) \to V_2(R)$ defined by $\{T(x,y,z)\} = (y-x, y-z)$ and also verify rank nullity theorem.

11424

B.Sc. IV Semester Degree Examination, May - 2018

MATHEMATICS

Differential Equations

Paper - 4.2

Time : 3 Hours

Maximum Marks : 60

Instructions to Candidates:

1) Answer **ALL** the questions section wise.

2) Mention the question numbers correctly.

Section - A

Answer any **TEN** of the following : **(10×2=20)**

1. Define order and degree of the differential equation

2. Solve $(1+X^2)dY+(1+Y^2)dX=0$

3. Show that the equation (12X+5Y-9) dX + (5X+2Y-4) dY = 0 is exact.

4. Solve $\frac{dY}{dX}+Y\cot X=4X\operatorname{cos}ecX$

5. Solve $P^2 - 5P - 6 = 0$

6. Solve the equation $(D^3-6D^2+11D-6)Y=0$

7. Solve the equation $(D^2+4D+4)Y=e^{2X}$

8. Evaluate $\frac{1}{D^2+a^2}.\cos aX$

9. Reduce the equation $[4X^2D^2+4XD-1]Y=4X^2$ and find the complementary function.

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10. Find a part of C.F. of the equation $X^2 Y_2 - (X^2 + 2X) Y_1 + (X + 2) Y = X^3 e^X$

11. S.T the equation $(2X^2 + 3X)\dfrac{d^2Y}{dX^2} + (6X + 3)\dfrac{dY}{dX} + 2Y = (X + 1)e^X$ is exact.

12. Find the Wronskian W for the equation $\dfrac{d^2Y}{dX^2} + Y = \cos ecX$

Section - B

Answer any **THREE** of the following : **(3×5=15)**

13. Solve $\dfrac{dY}{dX} = (4X + Y + 1)^2$

14. Solve the Bernaulli equation $\dfrac{dY}{dX} + Y \tan X = Y^3 \sec X$

15. Solve $X^2YdX - (X^3 + Y^3)dY = 0$ by finding integrating factor

16. Solve $Y = 2PX + Y^2P^3$

17. Find the general solution & singular solution of $X^2(Y - PX) = YP^2$ by using the substitution $X^2 = u, Y^2 = v$.

Part - C

Answer any **THREE** of the following : **(3×5=15)**

18. Solve $[D^2 + 4D + 4]Y = e^{2X} + e^{-2X}$

19. Solve $[D^2 - 2D + 1]Y = \cos 3X$

20. Solve $[D^2 - 10D + 16]Y = e^{4X}.\sin 2X$

21. Solve the simultaneous equations $\dfrac{dX}{dt} = 3X - Y; \dfrac{dY}{dt} = X + Y$

22. Solve $(X^2D^2 + 7XD + 5)Y = 2X^4$.

Section - D

Answer any **Two** of the following : **(2×5=10)**

23. Solve $X^2\dfrac{d^2Y}{dX^2}+(1-X)\dfrac{dY}{dX}-Y=e^X$ by finding a part of C.F.

24. Solve $Y''+(2\cos X+\tan X)Y'+\cos^2 X.Y=\cos^4 X$ by changing the independent variable.

25. Solve $Y''+a^2Y=\sec 3X$ by the method of variation of parameters.

26. Solve $\dfrac{d^2Y}{dX^2}-2\tan X\dfrac{dY}{dX}+5Y=e^X\sec X$ by changing the dependent variable.
