13DD 44 – V (45)

B.Sc. V Semester Degree Examination, Nov./Dec. 2013
MATHEMATICS
5.1 : Vector Analysis and Laplace Transformation

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** the Sections.*
*2) Write the question numbers **correctly**.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Find the directional derivative of $Q(x, y, z) = xy^2 + yz^3$ at the point (2, –1, 1).
2. Show that the vector $\vec{f} = (\sin y + z)i + (x\cos y - z)j + (x - y)k$ is irrotational.
3. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ and $|\nabla\phi|$ at (1, –1, 1).
4. If $f = (y^2 + z^2 - x^2)\, i + (z^2 + x^2 - y^2)\, j + (x^2 + y^2 - z^2)k$ find div f and curl f.
5. Define Fourier series of a function of period 2π.
6. Find Fourier co-efficient a_n if the function $f(x) = x$ in $(-\pi, \pi)$.
7. Find Fourier co-efficient b_n of $f(x) = e^{-ax}$ where $-\pi \le x \le \pi$, f(x) is periodic function with 2π.
8. Find the Laplace transform of $5e^{3t} + 6t^3 - 3\sin 4t + 2\cos 2t$.
9. Find :
   a) $L[\sin^2 t]$
   b) $L[6^t]$.
10. Find the inverse Laplace transform of $\dfrac{3+4s}{9s^2-16}$.
11. Find $L^{-1}\left[\dfrac{s+1}{s^2+2s-8}\right]$.
12. Define Dirac delta function.

P.T.O.

SECTION – B

Answer **any five** of the following : **(5×6=30)**

1. Prove that grad $(f \circ g) = f \times \text{curl } g + g \times \text{curl } f + (f.\nabla)g + (g.\nabla)f$.

2. Express the vector

   $f = 3yi + x^2 j - z^2 k$ in terms of spherical co-ordinates.

3. Verify Green's theorem for $\oint_c [(3x^2 - 8y^2)dx + 2y(2-3x)dy]$ where c is the boundary of rectangle enclosed by the lines $x = 0, x = 1, y = 0, y = 2$.

4. Verify Gauss divergence theorem for $F = 2xyi + yz^2 j + xzk$ and S is the total surface of the rectangular parallelopiped bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 2, z = 3$.

5. Construct the Fourier series for the function f(x) given by

   $$f(x) = \begin{cases} -K & -\pi < x < 0 \\ K & 0 < x < \pi \end{cases} \text{ and } f(x + 2\pi) = f(x).$$

6. Obtain the Fourier series of

   $$f(x) = \begin{cases} x & \frac{-\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x \le \frac{3\pi}{2} \end{cases}$$

   and $f(x + 2\pi) = f(x)$.

7. Represent the function f(x) as a Fourier cosine half range series

   $$f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$

SECTION – C

Answer **any five** of the following. **(5×6=30)**

1. Show that $L[t^n] = \frac{n!}{S^{n+1}}$ and hence find $L(t^3 + 3t^2 - 6t + 8]$.

2. Prove that $L[f(t)] = \frac{1}{1-e^{-ST}} \int_0^T e^{-st} f(t)\,dt$

   where f(t) is periodic function with period T.

3. Verify convolution theorem for $f(t) = t$, $g(t) = e^t$.

4. Find $L^{-1}\left[\frac{s}{(s-3)(s^2+4)}\right]$.

5. Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ given $y(0) = 0$ and $y'(0) = 1$.

6. Evaluate $L^{-1}\left\{\frac{5e^{-3s}}{s} - \frac{e^{-s}}{s}\right\}$.

7. Express the function interm of unit step function and find their Laplace transforms.

   $$f(t) = \begin{cases} 2t & 0 < t < \pi \\ 1 & t > \pi \end{cases}.$$

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B.Sc. V Semester Degree Examination, November/December 2013
Paper 5.2 : MATHEMATICS
Series Solutions, Total Differential Equations and Partial Differential Equations

Time : 3 Hours Max. Marks :80

Instruction : *Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Using generating function for Legendre's polynomial $P_n(x)$, prove that $P_n(1) = 1$.

2. Expand $f(x) = x^3$ in Legendre's series of the form $\sum_{n=0}^{\infty} C_r P_r(x)$.

3. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

4. Show that $J_0'(x) = -J_1(x)$.

5. Show that $\int_a^b J_0(x).J_1(x)dx = \frac{1}{2}\left[J_0^2(a) - J_0^2(b)\right]$.

6. Test the integrability of the total differential equation $xdy - ydx - 2x^2zdz = 0$.

7. Solve the simultaneous equation $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy^2}$.

8. Form the partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

9. Solve $p^2q^3 = 1$.

P.T.O.

10. Solve $z = pq$.

11. Solve $yp + xq + pq = 0$.

12. Find the complete integral of $q = px + p^2$.

SECTION – B

Answer **any five** of the following : **(5×6=30)**

1. Show that $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2 - 1}$.

2. Using the generating function for Legendre polynomial show that

$$nP_n(x) = xP_n'(x) - P_{n-1}'(x).$$

3. Derive the recurrence formula $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

4. Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3 - x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$.

5. Verify the condition of integrability and solve the equation

$$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$$

6. Solve $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$.

7. Solve $\frac{adx}{(b - c)yz} = \frac{bdy}{(c - a)zx} = \frac{cdz}{(a - b)xy}$.

SECTION – C

Answer **any five** of the following : **(5×6=30)**

1. Solve the equation $x^2p + y^2q = z^2$ which is in the form $Pp + Qq = R$.
2. Solve $x^2p^2 + y^2q^2 = z^2$ by putting $u = \log x$, $v = \log y$, $w = \log z$.
3. Solve $4(1 + z^3) = 9z^4 pq$.
4. Solve $z(p + q) = \tan x + \tan y$ by putting $u = z^2/2$.
5. Find complete, singular and general integral of $p^3 + q^3 = 27z$.
6. Solve $p(1 + q) = qz$ by Charpits method.
7. Solve $pxy + pq + qy = yz$ by Charpits method.

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B.Sc. V Semester Degree Examination, November/December 2013

MATHEMATICS

Paper – 5.3 (b) : Graph Theory

Time : 3 Hours

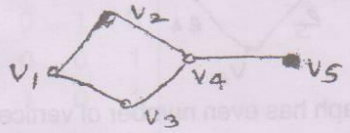
Max. Marks : 80

Instruction : *Answer **all** Sections.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Define graph and a loop.
2. Define multiple and pseudo graph.
3. Define complete graph and draw complete graph with 5-vertices.
4. Define regular graph and draw 3-regular graph with 4 vertices.
5. Define subgraph and spanning subgraph.
6. Define induced subgraph and write 2 induced subgraphs of the below graph.



7. Define isomorphic graphs and verify isomorphism for the following graphs.

8. Define path, trail and cycle in a graph.

P.T.O.

9. Define connected and disconnected graphs.

10. Define complementary and self-complementary graphs.

11. Define shortest path between two vertices.

12. Define adjacency and incidency matrices.

SECTION – B

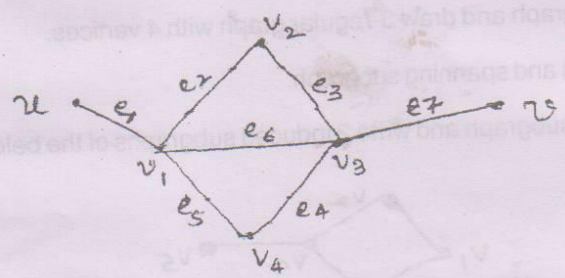
Answer **any five** of the following : **(5×6=30)**

1. In a G(p, q) graph prove that $\sum_{i=1}^{p} \deg v_i = 2q$ and in a graph the number of odd degree vertices are always even.

2. A graph G is regular iff $\Delta(G) = f(G)$.

3. In any graph G with 6 vertices either G or $\overline{G}$ contains a triangle.

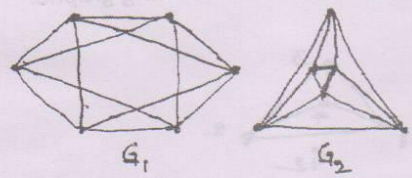
4. Write possible (a) walks (b) trails and (c) paths and (d) cycle in the following graph from u to v.



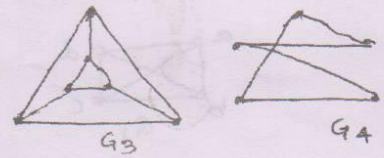
5. Prove that every cubic graph has even number of vertices.

6. Are the following graphs isomorphic ? Justify the answer with naming the vertices.

a)



b)



7. Define bipartite and complete bipartite graphs. And draw a complete bipartite star graph $(k_1, 6)$ of 6 vertices.

SECTION – C

Answer **any five** of the following : **(5×6=30)**

1. Every u – v walk contains a u – v path.

2. If G is a disconnected then $\overline{G}$ is connected.

3. Write Dijikstras algorithm for finding the shortest path.

4. Find the adjacency and incidency matrices of a 2-regular graph with 4-vertices.

5. Find adjacency and incidency matrices of a 2-regular graph with 5-vertices.

6. Find the graphs whose adjacency matrices are

i) $A(G_1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ ii) $A(G_2) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

7. Draw the graph to the following incidence matrix and also write its degree sequence.

$$\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{bmatrix}$$