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B.Sc. VI Semester Degree Examination, May/June 2017
MATHEMATICS
Paper – 6.1 : Numerical Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** the Sections.*
*2) Non-programmable scientific calculator may be **used**.*
*3) Write the question number **correctly**.*

SECTION – A

Answer **any ten** of the following. **(10×2=20)**

1. Find the absolute error, percentage error of the exact number $\frac{1}{3}$ whose approximate value is 0.333.

2. Find the relative error of the approximate number N = 437.4, if all its digits are valid.

3. State Newton-Raphson method for solving a non-linear equation f(x) = 0.

4. Evaluate : $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$.

5. Show that $y_3 = y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + \Delta^3 y_0$.

6. Express $x^4 - 12x^3 + 24x^2 - 30x + 9$ in factorial notation.

7. State Lagrange's interpolation formula for inverse interpolation.

8. Find the value of cos10° if the following table gives the values of sinθ for different value of θ.

θ	0°	10°	20°	30°	40°
sin θ	0.0000	0.1736	0.3420	0.5000	0.6428

9. Show that $\left(\frac{\Delta^2}{E}\right)e^x \frac{Ee^x}{\Delta^2 e^x} = e^x$.

P.T.O.

10. By using Trapezoidal rule evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$.

11. State Picard's method for first order initial value problem.

12. State Runge-Kutta fourth order formula for first order initial value problem.

SECTION – B

Answer **any five** of the following. **(5×6=30)**

13. Find the number of trustworthy figures assuming that the numbers are correct to three significant figures.

    i) $(0.318)^3$

    ii) $(0.507)^4$.

14. Find a real root of the equation $x^3 - x - 1 = 0$ using bisection method upto five stages over the interval (1, 1.5).

15. Solve, $x^3 - 9x + 1 = 0$ over (2, 4) by Regula-Falsi method to find the real root upto three approximation.

16. Solve by Gauss-elimination method $2x + y + 4z = 16$, $3x + 2y + z = 10$, $x + 3y + 3z = 16$.

17. Solve by Jacobi-Iteration method $5x - y = 9$, $-x + 5y - z = 4$, $y - 5z = 6$.

18. Construct forward difference table of the polynomial $f(x) = x^3 + x^2 - 2x + 1$, find the value of the polynomial at $x = 6$ by extending the table.

19. Evaluate :

    i) $\Delta \tan^{-1} ax$

    ii) $\Delta (x^2 + e^x + 2)$

    iii) $\Delta^2 [(1 - x)(1 + 2x)(1 - 3x)]$.

SECTION – C

Answer **any five** of the following. **(5×6=30)**

20. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 54 from the following table :

x	50	51	52	53	54
y	3.6840	3.7084	3.7325	3.7563	3.7798

21. Use Simpson's $\frac{1}{3}^{rd}$ rule to evaluate $\int_0^6 \frac{dx^3}{(1+x)^2}$ correct to three places of decimal in steps of one unit.

22. Using Weddle's and Trapezoidal rule calculate $\int_4^{5.2} \log x \, dx$.

23. Solve $\frac{dy}{dx} = 1 + xy$ by Picard's method, given y = 0, when x = 0 upto third approximation and obtain y when x = 0.2.

24. Solve $\frac{dy}{dx} = x^2 + y$ by Euler's modified method where y = 0.94, when x = 0 for x = 0.1 correct to four places of decimals.

25. Find the value of y at x = 0.2 correct to four decimal places if y(x) satisfies $\frac{dy}{dx} = x - y^2$ with $x_0 = 0$ and $y_0 = 1$ by Taylor's series method.

26. Solve $\frac{dy}{dx} = 1 + \frac{y}{x}$, given that $x_0 = y_0 = 2$ for x = 2.2 taking h = 0.1 by using Runge-Kutta method upto first approximation.

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B.Sc. VI Semester Degree Examination, May/June 2017

MATHEMATICS

Paper – 6.2 : Trigonometry, Complex Analysis and Improper Integrals

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** the questions **Section**wise.*

*2) Mention the question numbers **correctly**.*

SECTION – A

Answer **any ten** of the following : **(10×2=20)**

1. Express $\cos 4\theta$ in terms of descending powers of $\sin\theta$.

2. Find the real and imaginary parts of $\cos(x + iy)$.

3. Prove the identity $\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$.

4. Find all the values of $\text{Log}\left(1+i\sqrt{3}\right)$.

5. Show that $\text{amp}(z - 1) = \pi/2$ represents a line parallel to imaginary axis.

6. Define analytic function and show that $f(z) = \sin z$ is analytic.

7. If f(z) is analytic function such that f(z) is always real then f(z) is constant.

8. Show that $\int_C \frac{dz}{z-a} = 2\pi i$.

9. Test the convergence of $\int_0^\infty e^{-mx}dx, m > 0$.

10. Prove that $\Gamma(n+1) = n\Gamma(n)$.

P.T.O.

11. Evaluate $\int_0^{\pi/2} \cos^6 \theta d\theta$ using Beta and Gamma functions.

12. Evaluate $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$.

SECTION – B

Answer **any five** of the following : **(5×6=30)**

13. State and prove Cauchy-Riemann equations in Cartesian form.

14. Find the analytic function f(z) whose real part is $u = e^x(x\sin y - y\cos y)$.

15. State and prove Cauchy's Integral theorem.

16. Show that $u = 2xy - 2x + 4y$ is harmonic and find its harmonic conjugate.

17. Evaluate $\int_C \frac{e^z}{z + i\pi} dz$ where

    i) C is $|z| = 2\pi$ and

    ii) $|z| = \pi/2$.

18. If $f(z) = u + iv$ is analytic show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

19. Evaluate $\int_C \frac{z}{(9 - z^2)(z + 1)} dz$ where C is $|z| = 2$.

SECTION – C

Answer **any five** of the following : **(5×6=30)**

20. Expand $\sin^7\theta$ in a series of sines of multiples of θ.

21. If $\tan[\log(x + iy)] = a + ib$ and $a^2 + b^2 \neq 1$ then show that

$$\tan[\log(x^2 + y^2)] = \frac{2a}{1 - a^2 - b^2}.$$

22. Sum the series cosA + cos3A + cos5A + ..n terms.

23. Show that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

24. Using the definition of Gamma function show that $\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$.

25. Show that $\displaystyle\int_0^1 \frac{x^2}{(1-x^4)^{1/2}}\,dx * \int_0^1 \frac{1}{(1+x^4)^{1/2}}\,dx = \frac{\pi}{4\sqrt{2}}$.

26. Express $\displaystyle\int_0^a x^4\sqrt{a^2 - x^2}\,dx$ in terms of Beta function and evaluate it.

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B.Sc. VI Semester Degree Examination, May/June 2017
MATHEMATICS
Paper No. – 6.3 : Graph Theory

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **all** questions.*
*2) Write the question numbers **correctly**.*

SECTION – A

I. Answer **any ten** of the following : **(10×2=20)**

1) Define edge connectivity and vertex connectivity.

2) Define cut set and cut vertices.

3) Give an example of a connected graph G containing a cut vertex V such that G-V has 4 components.

4) Find all cut vertices in the graph G.

G : 

5) Which of the following graphs are separable or nonseparable ?

i) P_7

ii) C_6

6) Draw all trees with 1, 2, 3 vertices.

7) Define binary tree with an examples.

8) Define rank and nullity of a graph.

9) Define rooted trees with example.

10) Define Eulerian trial and Eulerian cycle.

11) Define Hamiltonian path and Hamiltonian graph.

12) Draw all forests with 3-vertices.

P.T.O.

SECTION – B

II. Answer **any five** questions : **(5×6=30)**

13) Let G be a connected graph with atleast 3 vertices. Prove that G is a block if and only if any two vertices of G lie on a common cycle.

14) Which of the following graphs are tree ?

i) $K_{1,4} + e$

ii) $K_3 - e$

iii) $C_6 - v$

15) State and prove Whiteney's Inequality theorem.

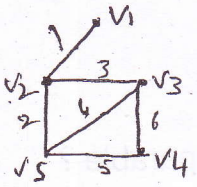
16) Prove that a Graph G then G is a cycle and P = q + 1.

17) Prove that if a graph G is connected, then G contains a spanning tree.

18) Prove that an edge of a graph G is bridge if and only if e is not on any cycle of G.

19) Draw all spanning trees of the graph G.

G :

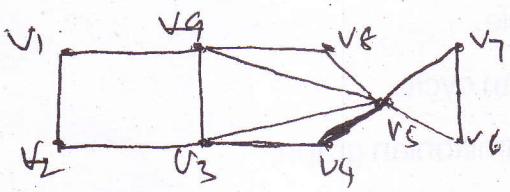


SECTION – C

III. Answer **any five** questions : **(5×6=30)**

20) Find an Eulerian cycle of the graph G shown below and a partition of edges of G into cycles.

G :



21) Construct a graph G with $K(G) = 2$, $\lambda(G) = 3$ and $\delta(G) = 4$.

22) Which of the following graphs are separable or non-separable.

i) $K_{1,4}$

ii) $K_{2,2}$

iii) $K_3 - e$

23) Give an example of a graph which is

a) Hamiltonian but not Eulerian

b) Eulerian but not Hamilton

c) Eulerian and also Hamilton

24) Prefix code and encode the following message "ROAD IS GOOD".

25) Prove that a connected graph G is Eulerian if and only if the set of edges of G can be partitioned into cycles.

26) Find three spanning trees of K_5.
